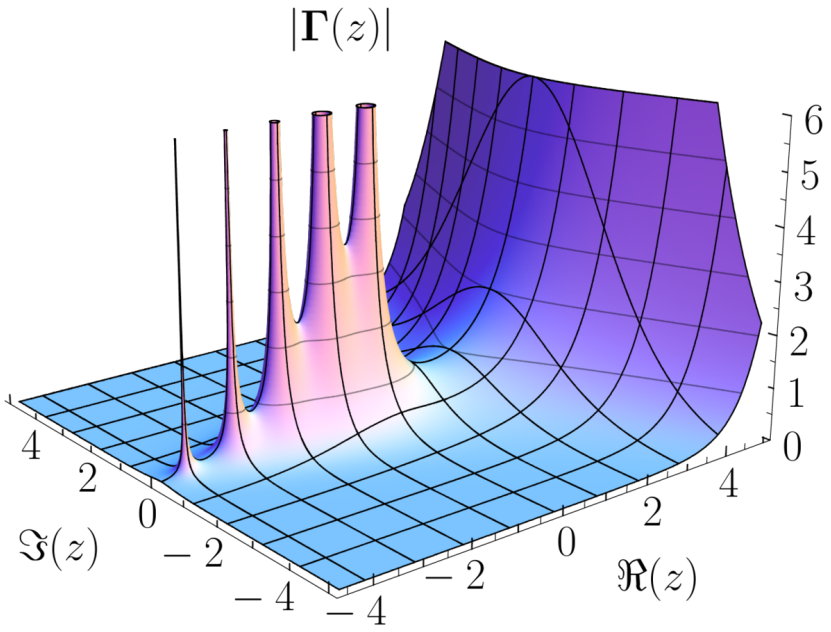
WILLIAM & MARY

DEPARTMENT OF MATHEMATICS

MATH 405
Complex Analysis

Fall 2022



The information below is given tentatively and subject to changes during the term. The course page can be found at `prclare.people.wm.edu/m405f22` *and will be updated regularly.*

General Organization

References:

We will not follow a particular textbook, but the standard texts below may be of use throughout the term:

- *Fundamentals of Complex Analysis, with applications to Engineering and Science*, by E. B. Saff and A.D. Snider
- *Real and Complex Analysis*, by W. Rudin
- *Complex Analysis*, by E. M. Stein and R. Shakarchi

Scheduled lectures: MW 3:30pm - 4:50pm in Boswell 302

Instructor: **Pierre Clare** (130 Jones Hall)

Office hours: Thursdays from 2pm to 3pm and by appointment or by chance.

Last day to add/drop	Sep. 12
Midterm examination 1	Oct. 17
Fall Break	Oct. 12 - 16
Last day to withdraw	Oct. 31
Midterm examination 2	Nov. 16
Thanksgiving Break	Nov. 23 - 27
Final examination	Dec. 15

Learning objectives

The purpose of this course is to introduce the basic concepts and techniques of analysis of functions of a complex variable. After discussing complex numbers and the geometry of the complex plane, we will focus on holomorphic functions and their properties: Cauchy-Riemann equations, harmonicity, integral representations and Cauchy's theorem, analyticity, the Maximum Modulus Principle, zeroes and poles, residue calculus and selected applications. Additional topics may include meromorphic extensions and applications to number theory, special functions or elements of index theory.

Outline of the course

I. **The complex plane**
   1. Complex numbers
   2. Geometry of the complex plane
   3. Series and power series
   4. The complex exponential and logarithm

II. **Functions of a complex variable**
   1. Analytic and holomorphic functions
   2. The Cauchy-Riemann relations

III. **Integral representations of functions**
   1. Curves and paths
   2. Curvilinear integrals
   3. The index
   4. Cauchy's theorem in convex domains
   5. Morera's theorem and the reflection principle

IV. **Analytic functions**
   1. The zeros of holomorphic functions
   2. Isolated singularities
   3. Cauchy estimates and Liouville's Theorem
   4. Limits of holomorphic functions
   5. Local behavior of holomorphic functions
   6. Meromorphy, residues and applications

Grades

The course grade will be based upon the scores on the midterm exams, the homework and the final exam, following Option A or Option B described below, whichever yields a higher score.

	Option A	Option B
Homework	20 %	20 %
Midterm 1	25 %	15 %
Midterm 2	25 %	15 %
Final	30 %	50 %

The final grade will be assigned according to the following scale:

$$100 \geq \textbf{A} \geq 93 > \textbf{A-} \geq 90$$

$$90 > \textbf{B+} \geq 87 > \textbf{B} \geq 83 > \textbf{B-} \geq 80$$

$$80 > \textbf{C+} \geq 77 > \textbf{C} \geq 73 > \textbf{C-} \geq 70$$

$$70 > \textbf{D+} \geq 67 > \textbf{D} \geq 63 > \textbf{D-} \geq 60 > \textbf{F}.$$

Attendance and Homework Policy

If you miss a lecture, it is your responsibility to inquire about the material covered in your absence.

Written homework will be assigned weekly. Extensions may (and usually will) be granted if requested at least twenty-four hours before the due date. ***Late homework will not be accepted.***

Academic Integrity

The **Honor Code** applies to all activities related to this course.

Cooperation on homework is permitted and encouraged, but if you work together, do not take any paper away with you. In other words, you can share your thoughts (say on a blackboard or draft paper), but you have to walk away with only your understanding. In particular, you must write the solution up on your own. You must acknowledge any cooperative work by adding a mention such as *Joint work with* Collaborator$_1$,... *and* Collaborator$_n$ right below your name on the front page.

On exams, you may not give or receive help from anyone. Exams in this course are closed book, and no notes, calculators or other electronic devices are permitted. The **Honor System** is responsible for resolving any suspected violations of the Honor Code. Do not hesitate to ask the instructor beforehand if you have any questions as to whether some action would be compatible with the Honor Code.

ADA Considerations

Students with disabilities who will be taking this course and may need disability-related accommodations are encouraged to make an appointment to see their instructor as soon as possible.

William & Mary accommodates students with disabilities in accordance with federal laws and university policy. Any student who feels they may need an accommodation based on the impact of a learning, psychiatric, physical, or chronic health diagnosis should contact Student Accessibility Services staff at 757-221-2512 or at sas@wm.edu to determine if accommodations are warranted and to obtain an official letter of accommodation. Visit **wm.edu/sas** for more information.

Mental and Physical Well-Being

William & Mary recognizes that students juggle different responsibilities and can face challenges that make learning difficult. There are many resources available at W&M to help students navigate emotional/psychological, physical/medical, material/accessibility concerns.

Asking for help is a sign of courage and strength. If you or someone you know is experiencing any of these challenges, we encourage you to reach out to the following offices:

- For psychological/emotional stress, please consider reaching out to the W&M Counseling Center; (757) 221-3620, 240 Gooch Dr., 2nd floor. Services are free and confidential.
- For physical/medical concerns, please consider reaching out to the W&M Health Center; (757) 221-4386, 240 Gooch Drive.
- For additional support or resources, please contact the Dean of Students by submitting a Care Report online or by calling 757-221-2510, or by emailing: `deanofstudents@wm.edu`.

For a list of many other resources available to students, see Health and Wellness Resources for Students.

As your professor, I also encourage you to reach out to me if you are facing challenges inside or outside the classroom; I will guide you to appropriate resources on campus.

Schedule of Topics

Week	Date	Topic
1	8/31	Complex numbers
2	9/05	**Labor Day - No class**
	9/07	The complex plane
3	9/12	Series and power series
	9/14	More on power series
4	9/19	The complex exponential polar forms and arguments
	9/21	Polar forms and arguments
5	9/26	Logarithms, branches
	9/28	Holomorphic and analytic functions
6	10/03	Holomorphic and analytic functions
	10/05	The Cauchy-Riemann relations
7	10/10	Curves in the complex plane
	10/12	Integrals along curves
8	10/17	Midterm 1
	10/19	The index
9	10/24	The Cauchy-Goursat theorem
	10/26	Cauchy's theorem for convex domains
10	10/31	Taylor expansions, Morera's theorem
	11/02	The reflection principle
11	11/07	Zeros of holomorphic functions
	11/09	Isolated singularities
12	11/14	Liouville's Theorem
	11/16	Midterm 2
13	11/21	**No class - Extra time on 11/07, 11/14, 11/28**
	11/23	**Thanksgiving Break - No class**
14	11/28	The Riemann sphere, meromorphy
	11/30	Local behavior of holomorphic functions
15	12/05	The Open Mapping and Maximum Modulus theorems
	12/07	Residues and applications
$\star$	**12/15**	**Final examination**