March 30, 2022

PLAY WITH PERMUTATIONS

1. Snevily's Conjecture

This conjecture has been solved, so I just put the reference here.

Arsovski, Bodan. A proof of Snevily's conjecture. Israel J. Math. 182 (2011), 505–508.

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2. Bialostocki's Conjecture

(**Conjecture**) *Let n be an even positive integer. Let $a_1, ..., a_n$ and $b_1, ..., b_n \in \mathbb{Z}/n\mathbb{Z}$ be such that*

$$\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i = 0.$$

Then there exists a permutation $\sigma \in S_n$ such that

$$\sum_{i=1}^{n} a_i b_{\sigma(i)} = 0.$$

- The only progress I am aware of is this paper:

Guo, Song; Sun, Zhi-Wei. On Bialostocki's conjecture for zero-sum sequences. Acta Arith. 140 (2009), no. 4, 329–334.

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3. Sun's Conjecture

(**Conjecture**) Let n be a positive integer, and $a_1, a_2, ..., a_n$ be a list of distinct real numbers. We can reorder the list as $b_1, b_2, b_3, ..., b_n$ such that the following n numbers

$$|b_1 - 0|,\ |b_2 - b_1|,\ |b_3 - b_2|,\ ...,\ |b_n - b_{n-1}|$$

are pairwise distinct.

- The only progress I am aware of is this paper:

Monopoli, Francesco. Absolute differences along Hamiltonian paths. Electron. J. Combin. 22 (2015), no. 3, Paper 3.20, 8 pp.

- See also this link for the problem (whether $c(n)$ is nonzero) I posted on MathOverflow:

https://mathoverflow.net/questions/385398/a-vandermonde-type-of-determinants-summed-over-permutations