# Research Workshop:
# Branching Problems and Symmetry Breaking

**Organizer:** Michael Pevzner (University of Reims)

June 10-11, 2021

In Representation Theory, branching problems ask how a given irreducible representation $\pi$ of a group $G$ behaves when restricted to subgroups $G' \subset G$. The decomposition of the tensor product of two irreducible representations (fusion rule) is a special case of this problem, where $(G, G')$ is of the form $(G_1 \times G_1, \Delta(G_1))$. In the general setting where $(G, G')$ is a pair of reductive groups and $\pi$ is an infinite dimensional representation of $G$, branching problems include various important situations such as theta correspondence and the Gross–Prasad–Gan conjecture, and branching laws may involve "wild behaviors" such as infinite multiplicities and continuous spectrum.

This workshop is devoted to recent progress in this area, with a particular emphasis on new analytical methods.

## Speakers

- Kazuki Kannaka (RIKEN iTHEMS)
- Toshihisa Kubo (Ryukoku U.)
- Quentin Labriet (U. of Reims)
- Ryosuke Nakahama (Kyushu U.)
- Ethan Shelburne (William & Mary / U. of British Columbia)
- Clemens Weiske (Paderborn U.)
- Genkai Zhang (Chalmers U.)

## Schedule

| JUNE 10 | | |
|---|---|---|
| 14:00 CET<br>21:00 JST | T. Kubo | *Differential symmetry breaking operators for* $(O(n+1,1), O(n,1))$ *on differential forms* |
| 14:45 CET<br>21:45 JST | R. Nakahama | *Computation of weighted Bergman inner products on bounded symmetric domains for* $SU(r,r)$ *and restriction to subgroups* |
| Tea time | | |
| 16:00 CET<br>23:00 JST | Q. Labriet | *Symmetry breaking operators and orthogonal polynomials* |
| 16:45 CET<br>23:45 JST | E. Shelburne | *Toward a holographic transform for the quantum Clebsch-Gordan Formula* |

| JUNE 11 | | |
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| 14:00 CET<br>21:00 JST | K. Kannaka | *The multiplicities of stable eigenvalues on compact anti-de Sitter 3-manifolds* |
| 14:45 CET<br>21:45 JST | C. Weiske | *Analytic continuation of unitary branching laws for real reductive groups* |
| Tea time | | |
| 16:00 CET<br>23:00 JST | G. Zhang | *Induced representations of Hermitian Lie groups from Heisenberg parabolic subgroups* |

## Abstracts

### Thursday, June 10

14:00 CET
**T. Kubo** (Ryukoku U.)
*Differential symmetry breaking operators for* $(O(n+1,1), O(n,1))$ *on differential forms*

Let $X$ be a smooth manifold and $Y$ a smooth submanifold of $X$. Take $G' \subset G$ to be a pair of Lie groups that act on $Y \subset X$, respectively. We call a differential operator $D$ between

the spaces of smooth sections for a $G$-equivariant vector bundle over $X$ and that for a $G'$-equivariant vector bundle over $Y$ a differential symmetry breaking operator (differential SBO for short) if $D$ is $G'$-intertwining.

In [Kobayashi-Kubo-Pevzner, Lecture Notes in Math. 2170], we classified all the differential SBOs from the space of differential $i$-forms over the standard Riemann sphere to that of differential $j$-forms over the totally geodesic hypersphere. In this talk we shall discuss how we classify such operators. This is a joint work with T. Kobayashi and M. Pevzner.

14:45 CET
**R. Nakahama** (Kyushu U.)
*Computation of weighted Bergman inner products on bounded symmetric domains for $SU(r,r)$ and restriction to subgroups*

Let $D \subset M(r,\mathbb{C})$ be the bounded symmetric domain, and we consider the weighted Bergman space $\mathscr{H}_\lambda(D)$ on $D$. Then $SU(r,r)$ acts unitarily on $\mathscr{H}_\lambda(D)$. In this talk, we compute explicitly the inner products for some polynomials on $\mathrm{Alt}(r,\mathbb{C})$, $\mathrm{Sym}(r,\mathbb{C}) \subset M(r,\mathbb{C})$, and prove that the inner products are given by multivariate hypergeometric polynomials when the polynomials are some powers of the determinants or the Pfaffians. As an application, we present the results on the construction of symmetry breaking operators from $SU(r,r)$ to $Sp(r,\mathbb{R})$ or $SO^*(2r)$.

16:00 CET
**Q. Labriet** (U. Reims)
*Symmetry breaking operators and orthogonal polynomials*

Symmetry breaking operators are intertwining operators for the restriction of an irreducible representation. In some cases, these are given by differential operators whose symbol is related to some classical orthogonal polynomials. First, I will describe the example of the Rankin-Cohen brackets which are symmetry breaking operators for the tensor product of two representations of the holomorphic discrete series of $SL_2(\mathbb{R})$. I will explain how they are related to Jacobi polynomials, and to the classical Jacobi transform. In a second part I will describe a link between orthogonal polynomials on the simplex and symmetry breaking operators for the tensor product of multiple holomorphic discrete series of $SL_2(\mathbb{R})$.

16:45 CET
**E. Shelburne** (William & Mary / U. of British Columbia)
*Toward a Holographic Transform for the Quantum Clebsch-Gordan Formula*

A holographic transform is an equivariant map which increases the number of variables in its domain, a space of functions. The tensor product of two finite dimensional irreducible representations of the Lie algebra $\mathfrak{sl}(2)$ decomposes into a direct sum of irreducible modules. In fact, the tensor product of representations of $\mathscr{U}_q(\mathfrak{sl}(2))$, the quantum analogue of $\mathfrak{sl}(2)$, decomposes in the same way. The purpose of this talk will be discussing the search for explicit holographic transforms associated with these decompositions.

## Friday, June 10

14:00 CET
**K. Kannaka** (RIKEN iTHEMS)
*The multiplicities of stable eigenvalues on compact anti-de Sitter 3-manifolds*

A *pseudo-Riemannian locally symmetric space* is the quotient manifold $\Gamma\backslash G/H$ of a semisimple symmetric space $G/H$ by a discontinuous group $\Gamma$. Toshiyuki Kobayashi initiated the study of spectral analysis of *intrinsic differential operators* (such as the Laplacian) of a pseudo-Rimannian locally symmetric space. Unlike the classical Riemannian setting, the Laplacian of a pseudo-Rimannian locally symmetric space is no longer an elliptic differential operator. In its spectral analysis, new phenomena different from those in the Riemannian setting have been discovered in recent years, following pioneering works by Kassel-Kobayashi. For instance, they studied the behavior of eigenvalues of intrinsic differential operators of $\Gamma\backslash G/H$ when deforming a discontinuous group $\Gamma$. As a special case, they found infinitely many *stable eigenvalues* of the (hyperbolic) Laplacian of a compact anti-de Sitter 3-manifold $\Gamma\backslash \mathrm{SO}(2,2)/\mathrm{SO}(2,1)$ ([Adv. Math. 2016]). In this talk, I would like to explain recent results about the *multiplicities* of stable eigenvalues in the anti-de Sitter setting.

14:45 CET
**C. Weiske** (Paderborn U.)
*Analytic continuation of unitary branching laws for real reductive groups*

Let $G$ be a real reductive group, $P$ a minimal parabolic and $H$ a reductive subgroup of $G$. Unitary branching laws describe how a unitary irreducible representation of $G$ decomposes into a direct integral of unitary irreducible representations of $H$ when restricted to the subgroup $H$. If the representation is in the unitary principal series and $H$ has an open orbit on the flag manifold $G/P$, Mackey theory reduces this problem to the Plancherel formula of a homogeneous space for $H$, which is known in many cases. In this case we show how to obtain branching laws for other unitary representations like complementary series representations from branching laws for the unitary principal series by analytic continuation. We focus on the exemplary case of the rank-one pair $(O(1,n+1),O(1,n))$.

16:00 CET
**G. Zhang** (Chalmers U.)
*Induced representations of Hermitian Lie groups from Heisenberg parabolic subgroups*

We study the induced representations of Hermitian Lie groups $G$ from Heisenberg parabolic subgroups $P$. We find the composition series and complementary series.