Useful Invention Or Absolute Truth: What Is Math?[1]

By G. Johnson

At the top of the list of science's unanswered questions, like what is consciousness and how did life begin, is the deepest mystery of all: Why does the universe appear to follow mathematical laws?

According to the Big Bang theory, matter, energy, space and time were created during the primeval explosion. Instantly, it seems, everything began unfolding according to a mathematical plan. But where did the mathematics come from? What are the origins of numbers and the relationships they obey?

The ancient followers of the Greek mathematician Pythagoras declared that numbers were the basic elements of the universe. Ever since, scientists have embraced a kind of mathematical creationism: God is a great mathematician, who declared, "Let there be numbers!" before getting around to "let there be light!"

Scientists usually use the notion of God metaphorically. But ultimately, most of them at least tacitly embrace the philosophy of Plato, who proposed, rather unscientifically, that numbers and mathematical laws are ethereal ideals, existing outside of space and time in a realm beyond the reach of humankind.

Because the whole point of science is to describe the universe without invoking the supernatural, the failure to explain rationally the "unreasonable effectiveness of mathematics," as the physicist Eugene Wigner once put it, is something of a scandal, an enormous gap in human understanding.

"We refuse to face this embarrassment," Reuben Hersh, a mathematician emeritus of the University of New Mexico in Albuquerque, wrote in his recent book, "What Is Mathematics Really?" (Oxford University Press, 1997). "Ideal entities independent of human consciousness violate the empiricism of modern science." While science is anchored in observations of the physical world, Dr. Hersh insists that mathematics is more of a human creation, like literature, religion or banking.

[1] New York Times, Feb. 10, 1998

Dr. Hersh's book is one of several recent works contending that mathematics is not an ethereal essence but comes from people who invented, not discovered it. The sentiments presented in the books are not entirely new and the mathematical puzzle has hardly been solved. But the idea of a human-centered mathematics may be gaining force and respect.

In ''The Number Sense: How the Mind Creates Mathematics'' (Oxford University Press, 1997), Stanislas Dehaene, a cognitive scientist at the National Institute of Health and Medical Research in Paris, marshals experimental evidence to show that the brains of humans -- and even of chimpanzees and rats -- may come equipped at birth with an innate, wired-in aptitude for mathematics. Gregory J. Chaitin, a mathematician at I.B.M.'s Thomas J. Watson Research Center in Yorktown Heights, N.Y., takes an anti-Platonist stance in ''The Limits of Mathematics'' (Springer, 1997). Two Berkeley scientists, George Lakoff and Rafael E. Nunez, are working on a book tentatively called ''The Mathematical Body,'' contending that even the most abstract mathematical concepts arise from basic human experience -- from the way the body interacts with the world. They gave a preview of their ideas in a chapter of another book published last year: ''Mathematical Reasoning: Analogies, Metaphors and Images,'' edited by Lyn D. English (Erlbaum).

The authors are all working mathematicians and scientists, not postmodern critics viewing the territory from afar. They emphatically reject those who try to dismiss mathematics and science as arbitrary constructions, or white male Eurocentric folklore. But they are just as adamant in rejecting what most mathematicians and many scientists have come to take for granted: the Platonic creed.

''The normal notion of pure math is that mathematicians have some kind of direct pipeline to God's thoughts, to absolute truth,'' Dr. Chaitin wrote in ''The Limits of Mathematics.'' While scientific knowledge is tentative and subject to constant revision, mathematics is usually seen as eternal. But Dr. Chaitin called on his colleagues to abandon mathematical Platonism and adopt a ''quasi-empirical'' approach that treats mathematics as just another messy experimental science.

''Quasi-empirical,'' he said, ''means that math ain't that different from physics.'' This view is laid out in detail in a revised edition of ''New Directions in the Philosophy of Mathematics,'' edited by Thomas Tymoczco (Princeton University Press, 1998).

Leopold Kronecker, a 19th-century mathematician, once said: ''The integers were created by God; all else is the work of man.'' Albert Einstein, taking a different view of whole numbers, wrote that ''the series of integers is obviously an invention of the human mind, a self-created tool which simplifies the ordering of certain sensory experiences.''

In ''The Number Sense,'' Dr. Dehaene went even further. The integers -- the smallest ones, anyway -- are hard-wired into human nervous systems by evolution, along with a crude ability to add and subtract. Mathematics, he believes, is ''engraved in the very architecture of our brains.''

"Because we live in a world full of discrete and movable objects, it is very useful for us to be able to extract number," he argued in a recent forum published on the Internet (www.edge.org) by the Edge Foundation. "This can help us to track predators or to select the best foraging grounds, to mention only very obvious examples."

By studying brain-damaged patients who have lost basic number skills, Dr. Dehaene and others have tentatively traced this arithmetical module to an area of the brain called the inferior parietal cortex, a poorly understood location where visual, auditory and tactile signals converge. Scientists are intrigued by clues that this region is also involved in language processing and in distinguishing right from left. Mathematics is, after all, a kind of language intimately involved with using numbers to order space. The inferior parietal cortex also seems to be important for manual dexterity, and arithmetic begins with counting on the hands. Imaging experiments, in which people's brains are monitored as they calculate, point to the same region as a primitive number processor.

If this neurological calculator has indeed been bequeathed by evolution, then traces of it should be found in other species. In making his argument, Dr. Dehaene draws on experiments over the last few decades suggesting that even rats have a rudimentary number sense. The animals were taught to press lever A four times and then lever B to get food, or to press lever A when they heard a two-tone sequence and lever B when they heard an eight-tone sequence. (To insure that the rats were responding to the number of signals and not just to their duration, the two-tone sequence sometimes lasted longer than the eight-tone one.)

Even more striking were later experiments in which rats were first trained to associate lever A with two tones and lever B with four tones. Then they were taught to associate A with two flashes of light and B with four flashes. If the rats heard two tones and saw two flashes they learned to push B, not A. They seemed to have comprehended the notion that two plus two equals four.

The rats were not precise. Trained to press one lever four times, they often pressed it five or six times, expecting to be rewarded just the same, or they confused a seven-tone sequence with an eight-tone one. But the experiments support the notion of a primitive neurological number processor, even in rodents.

In other experiments, chimpanzees seemed to learn simple arithmetic. Given a choice between one tray with a pile of three chocolate chips and another pile of four and a second tray with piles of two and three chips, they chose the first tray with the most candy. But when the totals on the trays differed by only one chip, the chimps were less likely to make the discrimination. The number sense is approximate, not exact. More recent experiments on infants, using Mickey Mouse toys instead of chocolate chips, found signs of the same kind of rough numerical ability in babies less than 5 months old.

Dr. Dehaene says this instinct is innate, as singing is for songbirds or spinning webs is for spiders. Numbers are not Platonic ideals but neurological creations, artifacts of the way the brain parses the world. In that sense they are like colors. Red apples are not inherently red. They reflect light at wavelengths that the brain, as it was wired by evolution, interprets as red.

While people are born with an understanding of the rudiments of arithmetic, he contends, going beyond that requires learning and creativity. Multiplication, division and the whole superstructure of higher mathematics -- from algebra and trigonometry, to calculus, fractal geometry and beyond -- are a beautiful improvisation, the work of human culture.

The ability to weave simple ideas, like two plus two equals four, into the tapestries of higher mathematics, he suggests, is not unlike the human skill for language. People take a relatively small collection of words and, using a few simple rules of grammar and syntax, create literature.

At the University of California at Berkeley, Dr. Lakoff, a linguist and cognitive scientist, and Dr. Nunez, a developmental psychologist, contend that the source of mathematics lies not just in the brain but in the human body and the physical world. People favor number systems based on 10 because they have 10 fingers and 10 toes. But that is just the beginning of the story.

Driven by a built-in number sense, the theory goes, primitive people explored the wonders of counting by playing with their fingers or putting rocks in a pile. But they found that counting could also be thought of as taking steps along a line to measure distance. That metaphor eventually allowed for the invention of more abstract concepts. Walk one way and you get the positive integers; walk the other way and you get the negative integers. The starting point is zero.

Multiplication by a positive number can be thought of as stretching; multiplying by a negative number makes something shrink.

Dr. Lakoff and Dr. Nunez call these ''grounding metaphors.'' In inventing mathematics, they contend, people also used ''linking metaphors'' to connect two sets of ideas. The sequence of numbers can be mapped onto the notion of a line. Now numbers are not fingers or rocks but points. Put two lines together at right angles and you get what mathematicians call a Cartesian plane, a two-dimensional graph that opens up a whole new arena to play in.

And so, floor by floor, the tower of mathematics is built. ''Students never learn that mathematics is a creative endeavor,'' Dr. Lakoff said in a recent interview. ''Mathematics is more glorious because it is humanly constructed.'' There is no such thing as pure mathematics or pure thought, he said -- they are physical activities.

That does not mean that mathematics is a relativistic free-for-all. The most basic mathematical inventions are rooted in the brain and body. Even mathematicians' loftier elaborations are tested against the universe. Of the infinite range of mathematical creations, scientists keep those that help them explain and predict reality. Mathematicians savor the others as ends in themselves, like paintings or symphonies.

But many scientists and mathematicians still doubt that evolution -- biological or cultural -- can adequately explain why mathematics works so well in describing the fundamental laws of the universe.

"Our ability to discover, and describe mathematically, Newton's equations has no immediate survival value," said Dr. Paul Davies, professor of mathematical physics at the University of Adelaide in Australia. "This point has even greater force when it comes to, say, quantum mechanics. The reason people find it hard to understand quantum physics is precisely because there is no survival value in being able to do so."

The reason mathematics is so effective, he says, remains a deep mystery. "No feature of this uncanny 'tuning' of the human mind to the workings of nature is more striking than mathematics," he wrote in "The Mind of God: The Scientific Basis for a Rational World" (Simon & Schuster, 1992).

Some hold out vague hopes that the mystery might be solved if humans ever encounter an alien civilization. If mathematics is indeed universal and eternal, the theory goes, then the aliens would understand concepts like pi, the ratio of a circle's circumference to its diameter. The Platonists' assume that there is "pi in the sky," as the British astronomer John D. Barrow said in a book by that name (Oxford University Press, 1992).

The anti-Platonists say there is no reason to believe the aliens would understand mathematical inventions from Earth. "The Platonist claim that every intelligence must produce prime numbers, pi and the continuum hypothesis is an example of simple anthropomorphism," Dr. Hersh said.

But if earthlings were utterly baffled by extraterrestrial mathematics, would the anti-Platonists have proved their point? Not necessarily.

"Alien intelligences may be so far advanced that their math would simply be too hard for us to grasp," Dr. Davies said. "The calculus would have baffled Pythagoras, but with suitable tuition he would have accepted it."

But what if the humans and the aliens could communicate mathematically? Would that decide the issue in favor of the Platonists? Not really.

"If the alien species had evolved in an environment similar to ours -- say, a world composed of distinct, movable objects -- then most likely its brain would have incorporated, through natural selection, the same regularities about the external world as we have," Dr. Dehaene said. "Thus, it would have a very similar arithmetic and geometry.

"But now, suppose that the alien species has evolved in a radically different environment, like a fluid world," he continued. "Then knowledge of movable objects would not be essential to its survival, while knowledge of fluid mechanics, vortices, etc. would be. I believe that this hypothetical species would have internalized in its brain regularities strikingly different from ours. Hence it would have radically different mathematics."

And so the argument continues to churn.

Several years ago, the French mathematician Alain Connes, arguing for the Platonists, and the French neurobiologist Jean-Pierre Changeux, taking the opposite side, tried to settle the matter with a debate. The result, translated and edited by M. B. DeBevoise, was the book "Conversations on Mind, Matter and Mathematics" (Princeton University Press, 1995).

Ranging over a vast field of topics including relativity, quantum mechanics, neurobiology, topology, game theory, information theory and non-Euclidean geometry, the two reached the end of their discussion with no resolution.

The best they could do was to agree to disagree.