Equitable Coloring in 1-Planar Graphs

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Joint with Daniel W. Cranston

Virginia Commonwealth University

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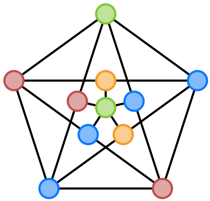

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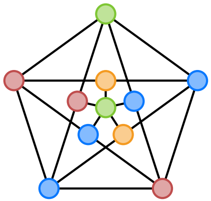

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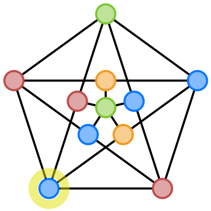

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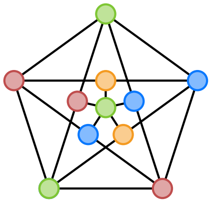

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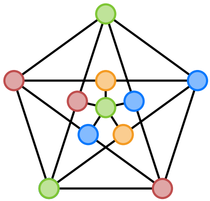

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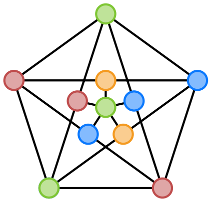

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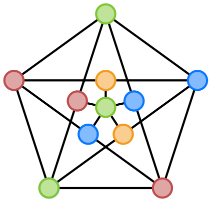

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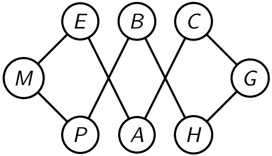
Courses	English, Biology, Chemistry, Math, Geography, Physics, Art, History



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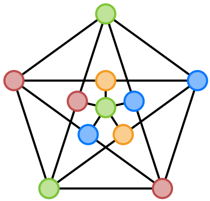

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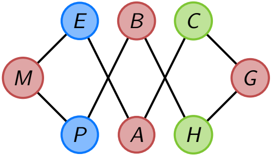
Geography
Art
Biology



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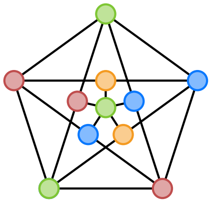

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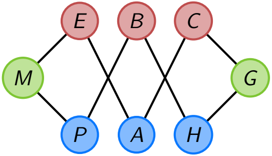

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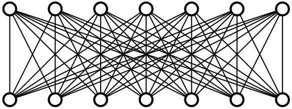

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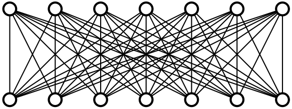

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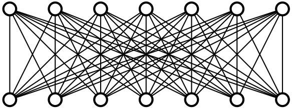

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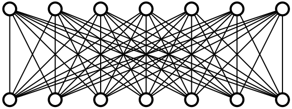

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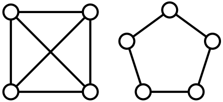

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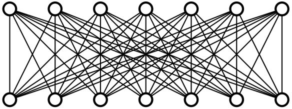

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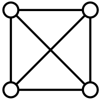

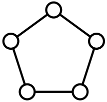

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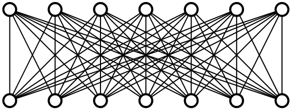

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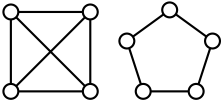

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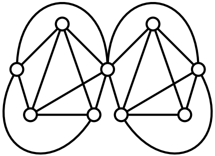
- CLW true for: Bipartite graphs (Lih-Wu 1996); planar graphs with $\Delta \geq 8$ (Kostochka-Lin-Xiang 2023, Nakprasit 2012, Yap-Zhang 1998); 1-planar graphs with $\Delta \geq 17$ (Zhang 2016)



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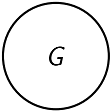

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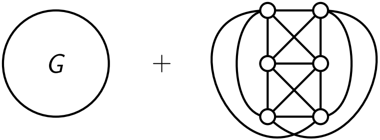

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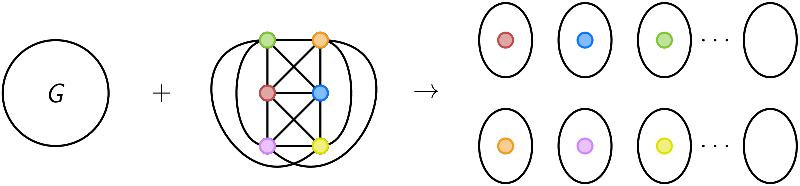

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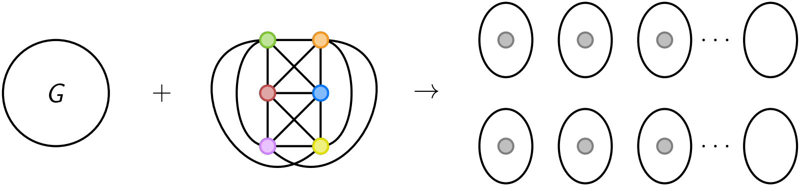

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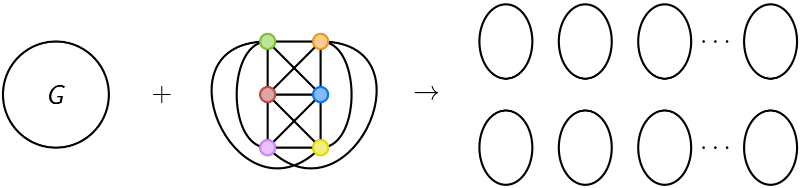

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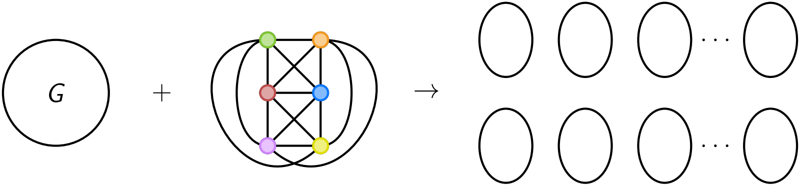

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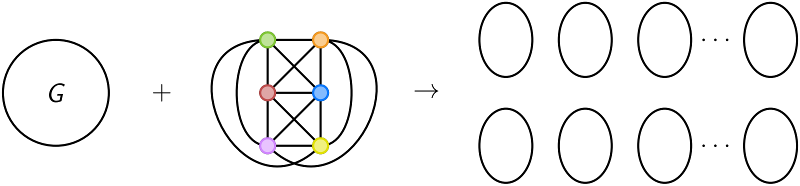

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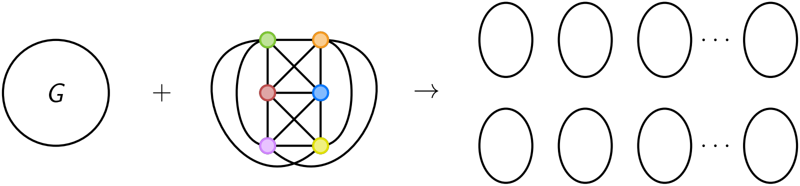

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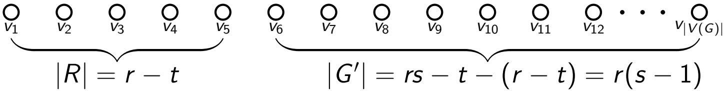

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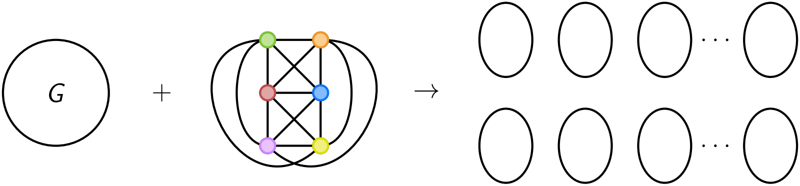

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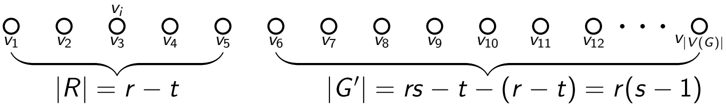

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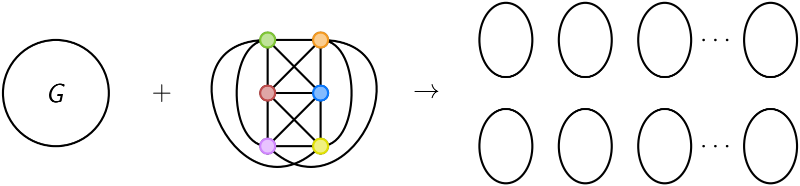

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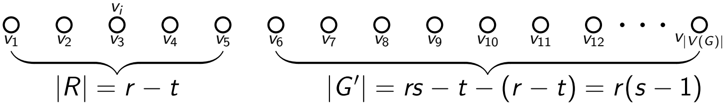

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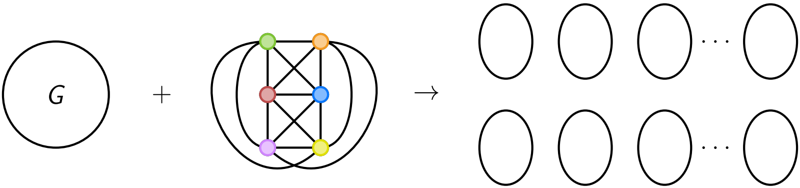
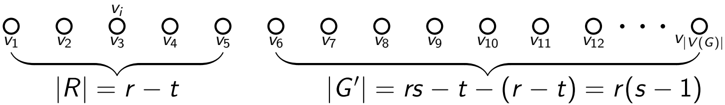


  - Case 2: $t \geq 7 \to$ 7-degenerate $\to$ every vertex has ≤ 7 neighbors ahead $\to$ delete first $r - t$ vertices $\to$ coloring vertex $i \to \#$ colors to avoid: $7 + r - t - i \leq 7 + r - 7 - 1$



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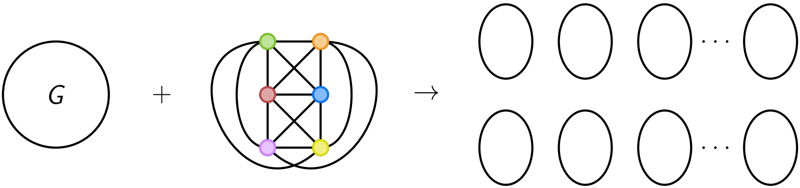

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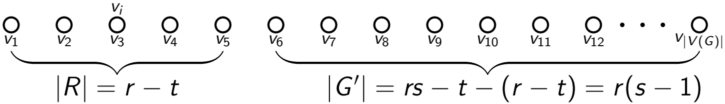

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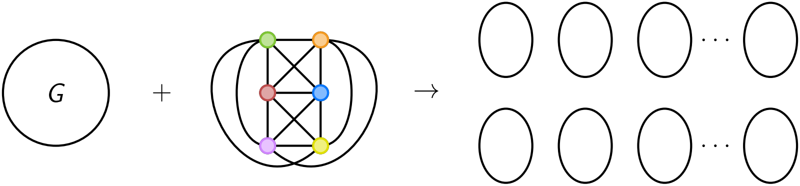

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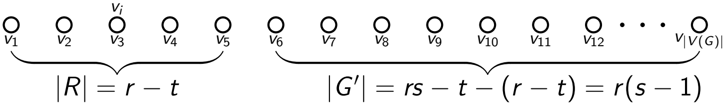

Proof Sketch (equitable r-coloring for $r \geq 13$)

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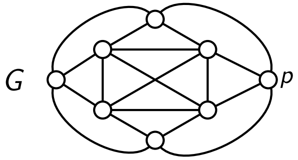

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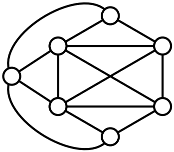

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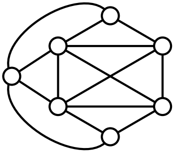

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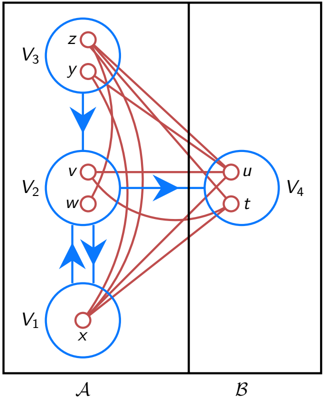

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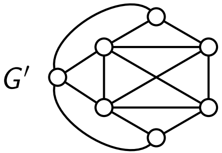

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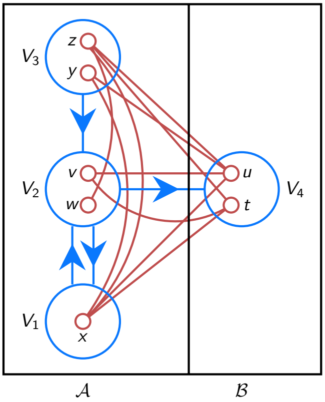

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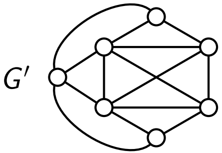

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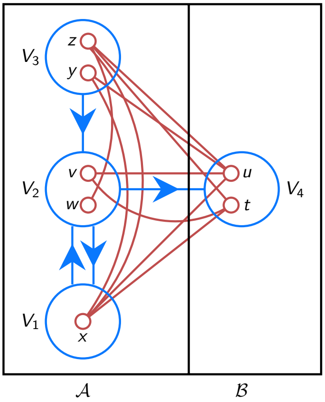

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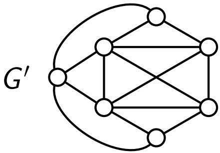

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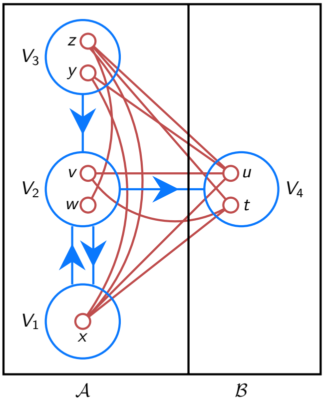

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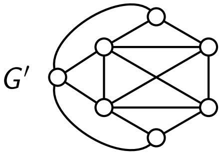

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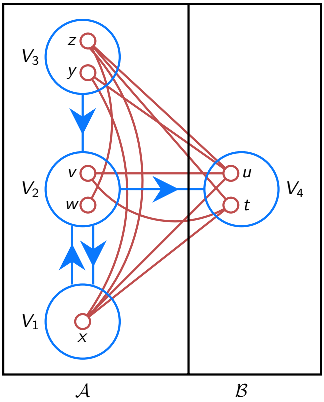

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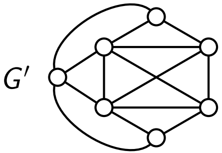

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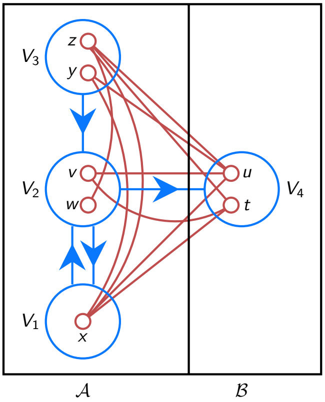

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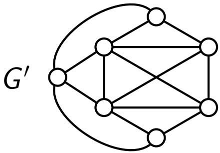

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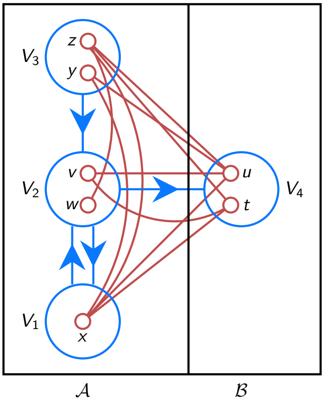

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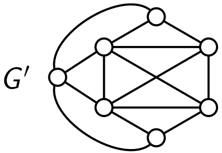

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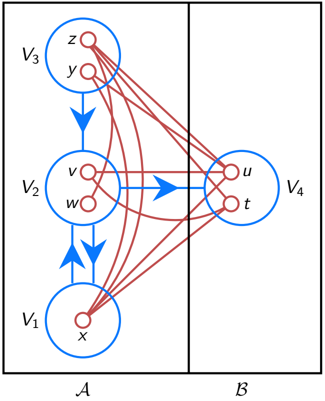

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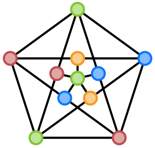

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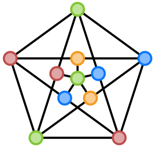

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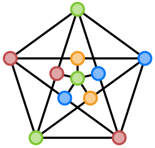

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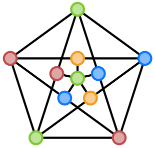

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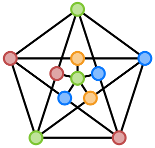

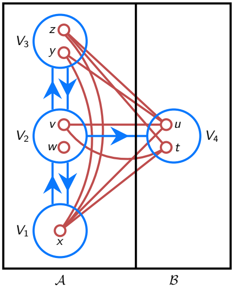

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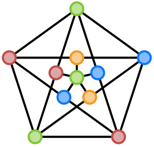

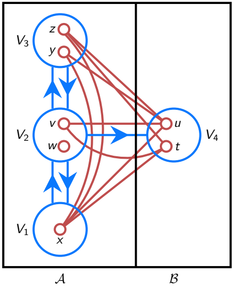

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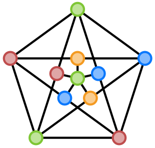

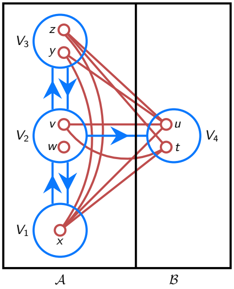

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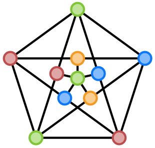

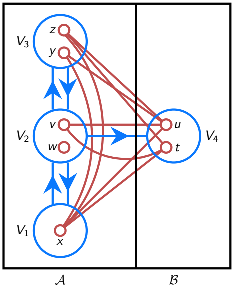

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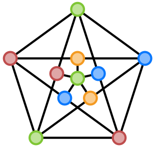

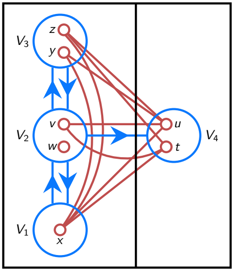

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